Upper Bound on Redundancy Reduction in Predictive-Entropy Subband Image Coding*

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Abstract This paper deals with the estimation of the theoretical and practical performance efficiency of a subband coding system for still images using memoryless quantizers optimized to entropy and mean-square error constraints. An approach with the concept of rate-distortion theory has allowed to derive a function in special form, which gives the performance gain in terms of signal-to-noise ratio and redundancy reduction. The theoretical upper bounds on entropy coded bit rate reduction were found to be 1.10; 1.51 and 2.10 bits per pixel for $M = 4$, 7 and 16 subbands, respectively. Some simulation results concerning practical performance are presented and interpreted on test images.

1 INTRODUCTION

Digital image compression is concerned with minimization of the number of bits used to represent an image. The aim of compression is to minimize the memory for storage or/and bandwidth for transmission. One of the promising methods for compression of digital image data is subband coding (SBC). The concept of SBC was first introduced in the context of speech coding in 1976. Since then, SBC technique has received considerable attention as an important source coding technique. The basic idea of SBC is to split up the frequency band of the signal and to code each subband separately using a coder and bit rate closely matched to the statistics of that particular band. As a result, aliasing will occur in the subbands and special care has to be taken to reconstruct the signal in order not to let the reconstruction suffer from aliasing error effects [1, 2]. Woods and O'Neil extended the fundamental SBC to the encoding of monochrome images in which a fullband image is split into 16 equally divided subbands by means of a two-dimensional (2-D) separable quadrature mirror filter (QMF) bank, each encoded by separate differential pulse code modulation (DPCM) encoder [3]. A bit allocation procedure

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is used to distribute the encoding bits among subbands in order to minimize the overall reconstruction mean-squared error (MSE).

In addition it was established in [3] that SBC has good subjective error properties and is appropriate for progressive image transmission (PIT). These encouraging results have led to a significant amount of research activity on subband image coding. In another paper Gharavi and Tabatabai have introduced a SBC scheme in which a QMF bank is used to split an image into 7 unequally divided subbands. The lowest frequency subband is encoded by DPCM and the others by zero-memory quantizers. Extensions to colour image coding were also considered in[4]. Results of incorporating vector quantization (VQ) in an SBC scheme were presented in [5,6]. In [7] the application of VQ to encode vectors consisting of samples from different subbands is presented. The coding gain of VQ over scalar quantization is computed analytically for the asymptotic case of high bit rates in [5].

Tanabe and Farvaradin presented two entropy-coded subband image coding schemes differing in the method of encoding the lowest frequency subband. A combined source channel coding scheme involving rate compatible convolutional codes is developed to overcome the sensitivity of two approaches to transmission noise[8].

One of the main goal in the image subband coding is to remove the vast amount of redundancy which exists in the spatial domain as well as in temporal direction[3]. Since most image coding methods are statistical in nature, the distortion criteria usually used are also statistical - the more commonly used being a normalized mean-squared error MSE or a signal-to-noise ratio (SNR).

The goal of this paper is to investigate subband coding technique in order to get an optimal coder performance in the sense of an upper bound on redundancy reduction for digital image applications when DPCM coding of the subbands is used. In keeping with this, subband coding strategy and DPCM as a method to encode subbands will be presented taking into account bit allocation procedure. In the second part, in order to obtain a theoretical lower bound on the bit rate required to encode each subband, a rate-distortion function R(D) will be derived for the DPCM case under the assumption of quantization optimized for Laplacian distributed noise. The theoretical upper bound of entropy coded bit rate reduction will be compared to the nonentropy DPCM coded case for a varying number of subbands. Finally, some numerical results will be presented. On the basis of some test images the validity of the the theory will be demonstrated, too.

2 SUBBAND CODING IMAGE COMPRESSION STRATEGY

One of the major advantages of SBC is that coding each subband separately can be done more accurately than coding the entire image. The type of coder for each subband must be determined by the individual subband statistics, while the actual bit rate for the coder is determined by the importance of that particular subband for the image at hand. Since each subband has a reduced bandwidth compared to the original fullband image, they may be decimated. This process of filtering and decimation is termed the analysis stage. Reconstruction is achieved by interpolation the decoded subbands applying appropriate filters and adding the reconstructed subbands together. This is the synthesis stage. The motivation for this approach is that the subbands can be encoded more efficiently than the original image.

The analysis and synthesis are usually accomplished by using a QMF bank because it is designed to eliminate the aliasing effects of bandpass filtering. Furthermore, 2-D QMFs can be separated into a cascade of two 1-D QMF s in each direction which makes the actual implementation of the filters quite easy [3,4]. The extension to splitting a multidimensional signal into subbands has been made by Vetterli, who also showed that the multidimensional QMF filtering problem could be solved by taking separable filters, thus reducing the problem again to one dimension [9]. By applying the QMF bank, we can split the original fullband image into four equally divided subbands referred to as "11", "12", "21" and "22" subbands. Here, the first number refers to row filtering and the second one pefers to column filtering. Also, "1" and "2" are used to denote "low" and "high" frequencies, respectively. Each of these subbands can be once more split into four equally-divided subbands by using the same QMF bank resulting in a total of 16 equally divided subbands. By parallel appllcation of a low pass filter and a high pass filter, the input signal spectrum is split into two overlapping subbands decimated by 2 : 1. An important factor in subband techniques is the relatively low complexity of the filters needed to split the signal into useful subbands .

When an image has been expressed in the fewest possible bits per pixel (bpp), it is said to be optimally encoded. In the image SBC advantage is taken of the nonuniform distribution of energy in the frequency domain to allocate the bits which represent the subband images. The division into frequency components removes the redundancy in the input and has the advantage that the number of bits used to encode each frequency band can be different, so that the encoding accuracy is always maintained at the required frequency bands.

SBC gives some properties like the continuous frequency analysis and reaches the limit of the rate distortion theory. These attractive advantages motivated our investigation of SBC. The distortion allocation problem is to minimize the encoder output entropy subject to a constraint on the MSE.

3　A BOUND OF SIGNAL TO QUANTIZING NOISE RATIO FOR DPCM SYSTEMS

The general image data compression problemme is achieving apriori trade-off between an acceptable level of distortion for a given bit rate. This trade-off is governed by the rate-distortion function. In the rate-distortion approach, the maximum allowable average distortion is constrained, while the required bit rate is minimized. On the other hand, in the distortion-rate approach, the rate is constrained and the resulting average distortion is minimized. It means that if the source is stationary, there exists a monotonically non-increasing distortion-rate function which provides a lower bound on the average distortion for a given rate, and hence an upper bound on the performance of practical image coders [10, 11] .

Assume that an image signal x arising from the source is expressed by sample values. The restriction is that the MSE per signal sample should not exceed D. When $x = (x_1, x_2, \ldots, x_L)^T$ is a L-dimensional random variable and has a sufficiently smooth density function $p(x)$, the rate-distortion function $R(D)$ is given by [12]

$$R(D) = \frac{L}{2}\log_2\frac{1}{D} + h(x) - \frac{L}{2}\log_2 2\pi e \quad (1)$$

where $h(x)$ is the differential entropy of x. The behaviour of $R(D)$ in L-dimensional space is determined in the first place by the dimension of the space, whereas the differential entropy $h(x)$ occurs only in the second term of the expression for $R(D)$.

When x is an one-dimensional random variable having the variance σ^2 and the Laplacian distribution, putting $h(x) = 1/2\log_2(2e^{\sigma 2})$ in the equ ation (1), wehave

$$R(D) = \frac{1}{2}\log_2\frac{\sigma^2}{D} - \frac{1}{2}\log_2\frac{\pi}{e} \quad (2)$$

Consider a DPCM system with entropy coding marked as EC-DPCM. A block diagram of the EC-DPCM encoder is shown in Fig. 1. The terms of the sequence of quantizing levels transmitted are statistically independent, but they are not equally likely. Entropy coding is a variable-length coding procedure. When the symbols to be transmitted are independent, it is possible to generate codes so that the average word length of these codes is approximately equal to the entropy of the symbols. To realize efficient coding when using entropy coding, it is necessary to keep the quantizing error fixed and obtain the quantization characteristics

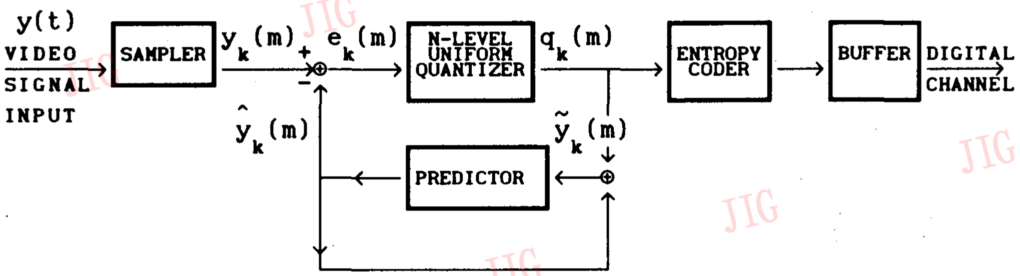

Fig. 1 DPCM system with entropy coding (EC-DPCM)

that will minimize the entropy. Panter and Dite have shown that minimum mean-square quantizing noise σ_q^2 is given by [13]

$$\sigma_q^2 = \frac{2}{3N^2}\left[\int_0^v p^{1/3}(x)\mathrm{d}x\right]^3 \tag{3}$$

where N is the total number of quantizing levels, the range of quantizer is $(-v,v)$, while $p(x)$ is an even function representing the Laplacian probability density function (pdf) of the input to the quantizer. Namely, for an input image, the amplitude density function of the quantizer input is approximately [14]

$$p(x) = \frac{1}{\sqrt{2}\,\sigma_e}\exp\left(-\frac{\sqrt{2}}{\sigma_e}|x|\right) \tag{4}$$

where σ_e is the r. m. s. value of the quantizer input. As v in equation (3) becomes large, and solving the integral, we obtain an approximation for the mean-square value of the quantizing noise to be

$$\sigma_q^2 = \frac{9}{2N^2}\sigma_e^2 \tag{5}$$

For the DPCM system the quantizer is designed to minimize σ_q^2/σ_e^2 for a fixed number of quantizing levels N. On the other side, for EC-DPCM system a quantizer is designed to minimize σ_q^2/σ_e^2 when the entropy of the quantizer output is fixed. For both DPCM systems, this results in minimizing σ_q^2/σ_e^2 and therefore maximizing the signal-to-noise ratio (SNR) for a fixed bit rate in the channel.

The bit rate C used to transmit the quantizing levels through a channel for the DPCM system without entropy coding is

$$C = 2f_m\log_2 N \tag{6}$$

where $2f_m$ is the sampling rate. Using equation (5) in equation (6), we obtain

$$C = f_m\log_2\frac{9}{2}\frac{\sigma_e^2}{\sigma_q^2} \tag{7}$$

For the EC-DPCM system, the bit rate in the channel is equal to the sampling ratemultiplied by the first-order entropy of the quantizer output H, i. e

$$C_1 = 2f_m H$$

Since the components of the quantizer output sequence are independent of each other and uncorrelated, the entropy of the quantizer output is

$$H \cong H_1 - H_q$$

where H_1 and H_q are the entropies of the quantizer input and quantizing noise sequence, respectively. Since the quantizing noise has a uniform pdf, it will be

$$H_q = -\int_{-\Delta/2}^{\Delta/2} 1/\Delta\log_2(1/\Delta)\mathrm{d}x = \log_2\Delta$$

Thus, when the number of quantizing levels gets large,

$$H \cong H_i - \log_2\Delta \tag{8}$$

where Δ represents the uniform quantizer's step size. For the uniform quantizer, as the number of levels N gets large, the quantizing noise becomes uncorrelatedwith the signal. The mean-square quantizing error is related to

$$\varepsilon_q^2 = \frac{1}{12}\Delta^2$$

For Laplacian quantizer input signals, using equation (4), we obtain

$$H_i = -\int_{-\infty}^{\infty} p(x)\log_2 p(x)\mathrm{d}x$$

$$=\log_2(\sqrt{2}\,e\sigma_e) \tag{9}$$

The first-order entropy of the quantizer output is

$$H = \frac{1}{2}\log_2 \frac{e^2}{6}\frac{\sigma_e^2}{\varepsilon_q^2} \tag{10}$$

while the bit rate is

$$C_1 = f_m \log_2 \frac{e^2}{6}\frac{\sigma_e^2}{\varepsilon_q^2} \tag{11}$$

We set equation (11) equal to equation (7) giving

$$\frac{9}{2}\frac{\sigma_e^2}{\sigma_q^2} = \frac{e^2}{6}\frac{\sigma_e^2}{\varepsilon_q^2}$$

i. e.

$$\varepsilon_q^2 \cong 0.27\sigma_q^2 \tag{12}$$

since σ_e^2 is the same for DPCM systems with and without entropy coding. Equation (12) gives the relationship between the quantizing noise powers, when the two systems operate at the same bit rate

$$\frac{\sigma^2}{\varepsilon_q^2} = \frac{27}{e^2}\frac{\sigma^2}{\sigma_q^2}$$

When the bit rate is large, a quantizing system using entropy coding achieves a signal-to-quantizing noise ratio SQNR of $10\log_{10}\frac{27}{e^2} = 5.6$ dB greater than a system without entropy coding.

If we assume a minimal bit rate in $R(D)$ sense of the coded quantizer output, the bit rate in the channel is equal to the sampling rate multiplied by the $R(D)$ of the quantizer output

$$C_2 = 2f_m R(D) \tag{13}$$

Using equation (2) and putting $\sigma = \sigma_e$, it will be

$$C_2 = f_m \log_2 \frac{e}{\pi}\frac{\sigma_e^2}{\varepsilon_q^2} \tag{14}$$

because using uniform quantization gives $\varepsilon_q^2 = D$, where ε_q^2 is the mean-squarequantizing error. To compare the quantizing noise of the system in which we assume the minimum bit rate in $R(D)$ sense of the quantizer output and the system without entropy coding, we set equation (14) equal to equation (7) giving

$$\frac{9}{2}\frac{\sigma_e^2}{\sigma_q^2} = \frac{e}{\pi}\frac{\sigma_e^2}{\varepsilon_q^2}$$

or

$$\varepsilon_q^2 \cong 0.19\sigma_q^2 \tag{15}$$

Since σ_e^2 is the same for both systems equation (15) gives the relationship between the quantizing noise powers when two systems operate on the same bit

$$\frac{\sigma^2}{\varepsilon_q^2} = \frac{9\pi}{2e}\frac{\sigma^2}{\sigma_q^2}$$

when the bit rate is large, a quantizing system using the minimum bit rate in $R(D)$ sense can give a SQNR of $10\log_{10}\frac{9\pi}{2e} = 7.2$ dB greater than a system without entropy coding [12].

4 BIT ALLOCATION IN A SUBBAND SYSTEM

Our use of DPCM as a method to encode the subbands is motivated by increased efficiency of a predictive encoder for a nonwhite power spectral density. The advantage of such a coding scheme is that the quantization noise generated in a particular band is limited largely to that band in reconstruction and is not allowed to spread to other bands. The overall subband coding system with DPCM is shown in Fig. 2. On the other hand, from Fig. 1 we can see that for DPCM, the reconstruction MSE is equal to the quantizer MSE, i. e

$$E\left[(y_k(m) - \tilde{y}_k(m))^2\right]$$
$$= E\left[(e_k(m) - q_k(m))\right]^2 = \sigma_{r,k}^2$$

where $\sigma_{r,k}^2$ represents the variance of the recon-

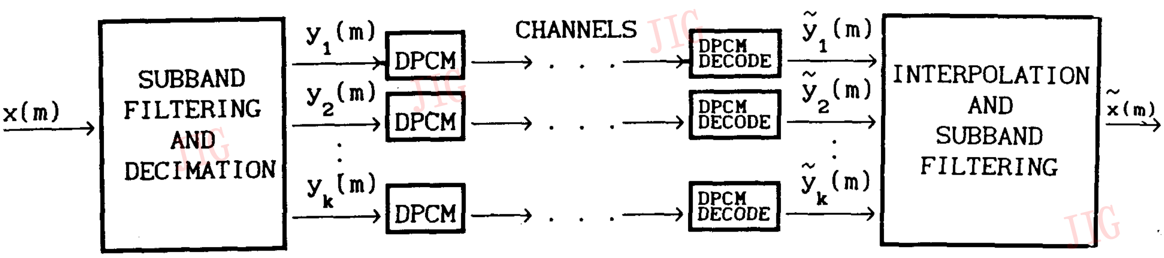

Fig. 2　Subband coding system with DPCM

struction error

$$r_k(m) = y_k(m) - \tilde{y}_k(m)$$

Consider now the case of M subbands of equal bandwith. If we assign B_k bits to subband k, we have the following average bit rate per subband

$$B = \frac{1}{M}\sum_{k=1}^{M} B_k \tag{16}$$

The overall MSE incurred in a subband coding scheme with M subbands is given by

$$D = \sum_{k=1}^{M} D_k(B_k)$$

in which $D_k(B_k)$ is used to denote the distortion rate performance of the encoder operating in the k-th subband at B_k bits per sample. We have assumed that all subbands have the same number of image-sample or picture element-pixels. If this is not the case, the $B_k' s$ should be multiplied by appropriate weighting coefficients to account for the variability in the number of pixels per subband. Denoting by $g(B_k)$ a slowly varying function of the number of bits assigned to subband k as well as the prediction error variance by $\sigma^2_{p,k}$, we have for the variance of the reconstruction error to be [15]

$$D_k(B_k) = \sigma^2_{r,k} = g(B_k)\sigma^2_{p,k}2^{-2B_k} \tag{17}$$

A bit allocation procedure is used to distribute the encoding bits among subbands in order to minimize the overall reconstruction MSE. The bit allocation between bands assumes that for a given average rate, the MSE between the source image and reconstructed one is minimized. For a given band spectrum, we achieve this goal by minimizing the accumulated distortion of all the bands. The value $g(B_k)$ in equation (17) equals 4.5 and is valid for the Laplacian pdf of the quantizer input. On the other side, the sum of the error variances of the M subbands is

$$\sum_{k=1}^{M}\sigma^2_{r,k} = \sum_{k=1}^{M} g(B_k)\sigma^2_{p,k}2^{-2B_k} \tag{18}$$

In what follows, we should minimize (18) subject to the constraint of (16). This solution can be obtained by using Lagrange multipliers. The MSE optimal bit assignment is [16]

$$B_k = B + \frac{1}{2}\log_2\left[\frac{\sigma^2_{p,k}}{\sigma^2_{gm}}\right], 1 \leqslant k \leqslant M \tag{19}$$

where

$$\sigma^2_{gm} = \left(\prod_{k=1}^{M}\sigma^2_{p,k}\right)^{1/M}$$

The minimum overall MSE is obtained to be [15]

$$\sigma^2_q = M\sigma^2_{gm}\varepsilon^2 2^{-2B} \tag{20}$$

where ε^2 is a constant quantizer performance factor. The result given by equation (19) assumes an approximate relationship between the MSE and the bit rate of each of the predictive quantizers, while $B_k' s$ must be nonnegative and integer.

5 A BOUND OF REDUNDANCY REDUCTION FOR SBC SYSTEM WITH DPCM

In order to achieve image compression as high as possible, we will use EC-DPCM for encoding subbands. Firstly, we take into account the first-order entropy of the quantizer output from the equation (10) and the allocated bit rat e from (19). It will be

$$\frac{1}{2}\log_2\frac{e^2}{6}\frac{\sigma^2_e}{\varepsilon^2_q} = B + \frac{1}{2}\log_2\frac{\sigma^2_{p,k}}{\sigma^2_{gm}}$$

Thus, the average bit rate in the EC-DPCM system becomes

$$B = \frac{1}{2}\log_2\frac{e^2}{6}\frac{\sigma^2_{gm}}{\varepsilon^2_q} \tag{21}$$

remembering $\sigma^2_e = \sigma^2_{p,k}$.

On the other hand, the minimum overall MSE obtained from equation (20) equals the mean-square value of the quantizing noise, i.e

$$\sigma^2_q = M\sigma^2_{gm}2^{-2B} \tag{22}$$

To satisfy our demand for high compression, we have from equations (21) and (22)

$$\frac{1}{2}\log_2\frac{e^2}{6}\frac{\sigma^2_q}{M\varepsilon^2_q} = 0$$

i.e

$$\frac{\sigma^2}{\varepsilon^2_q} = \frac{6M}{e^2}\frac{\sigma^2}{\sigma^2_q} \tag{23}$$

It can be concluded that using EC-DPCM to encode the subbands, we can achieve SQNR of $10\log_{10}\frac{6M}{e^2}$ [dB] greater than using PCM fullband method. The corresponding redundancy reduction in bits per pixel (sample of the image) is

$$R = \frac{\text{Improvement SQNR [dB]}}{6.02}$$

$$= \frac{1}{6.02} 10\, log_{10} \frac{6M}{e^2} \quad [\text{bpp}]$$

The Fig. 3, curve b) represents SQNR gain versus number of subbands M in EC-DPCM over fullband PCM coding.

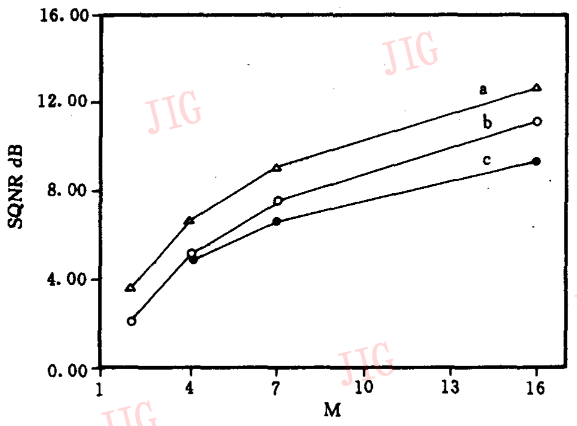

Fig. 3　*Signal to quantizing noise ratio (SQNR) gain versus the number of subbands M in*
SBC EC-DPCM coding over fullband DPCM coding:
△—— Upper bound in rate-distortion theory $R(D)$ sense (curve a),
□—— Theoretical result in SBC EC-DPCM coding, assuming the Laplacian distribution of subbands and ideal entropy coding of the independent individual pixels in subbands (curve b),
◇—— Simulation results in SBC EC-DPCM coding of test image LENA (curve c).

To obtain an upper bound on redundancy reduction, we have to assume theminimum bit rate in $R(D)$ sense of the quantizer output for each subband. Thus, we can write for the MSE optimal bit assignment using equations (2) and (19) that

$$\frac{1}{2}\log_2 \frac{e}{\pi} \frac{\sigma_e^2}{\varepsilon_q^2} 2 = B + \frac{1}{2}\log_2 \left[\frac{\sigma_{p,k}^2}{\sigma_{gm}^2}\right]$$

i . e

$$B = \frac{1}{2}\log \frac{e}{\pi} \frac{\sigma_{gm}^2}{\varepsilon_q^2} \qquad (24)$$

because $\sigma_e^2 = \sigma_{p,k}^2$.

Our redundancy reduction optimization problem includes the condition

$$\frac{1}{2}\log_2 \frac{e}{\pi} \frac{\sigma_q^2}{M\varepsilon_q^2} = 0$$

Thus, we have

$$\frac{\sigma^2}{\varepsilon_q^2} = \frac{M\pi}{\varepsilon} \frac{\sigma^2}{\sigma_q^2} \qquad (25)$$

It can be seen that for the DPCM system assuming the minimum bit rate in $R(D)$sense of the quantizer output for each subband, we obtain a SQNR of $10\log_{10}\frac{M\pi}{e}$[dB] greater than a system without entropy coding. This gain is an upper bound on SQNR for subband entropy coding. The corresponding upper bound on redundancy reduction in bits per pixel becomes

$$R = \frac{1}{6.02} 10\log_{10} \frac{M\pi}{e} [\text{bpp}]$$

The Fig. 3, curve a) shows the upper bound on SQNR gain versus number of subbands M. The complete numerical values of the gain in SQNR, redundancy reduction, the upper bound on SQNR gain as well as the corresponding redundancy reduction are easily carried out for 2, 4, 7, 8 and 16 subbands and included in Tab. 1.

As it can be seen, for a given number of subbands 2, 4, 7, 8 and 16, SQNR gain using EC-

Table. 1　Theoretical signal to quatizing noise ratio (SQNR) gain, redundancy reduction and their upper bounds in EC-DPCM subband image coding

NUMBER OF SUBBANDS M	SQNR GAIN [dB]	REDUNDANCY REDUCTION [bpp]	UPPER BOUND ON SQNR GAIN [dB]	UPPER BOUND ON REDUNDANCY REDUCTION [bpp]
2	2.11	0.35	3.64	0.60
4	5.12	0.85	6.65	1.10
7	7.52	1.25	9.09	1.51
8	8.13	1.35	9.66	1.60
16	11.14	1.85	12.67	2.10

DPCN to encode subbands increases from 2. 11 dB to 11. 14 dB over DPCM system. The corresponding bit rate savings are from 0. 35 to 1. 85 bits per pixel. On the other hand, assuming the minimum bit rate in R(D) sense of the quantizer output for each subband, we can achieve SQNR from 3. 64 dB to 12. 67 dB greater than in a DPCM system. This is an upper bound on SQNR gain. The corresponding upper bound on redundancy reduction is from 0. 60 bits per pixel to 2. 10 bits per pixel.

6 SIMULATION RESULTS

In order to verify our theoretical results, as well as to demonstrate practicalperformances concerning subband coding system with EC-DPCM, some computer simulations are carried out .

In our experiment the monochrome test image "Lena" with 256 gray scale levels and the resolution 256 × 256 pixels, shown in Fig. 4(a), was used. For this case, the first order entropy can be calculated as

$$H = -\sum_{k=0}^{255} p_k \log_2 p_k \quad [\text{bpp}] \qquad (26)$$

where p_k is the probability of a k-th gray scale level in an image.

Our major constraints are: memoryless uniform quantizer in DPCM loop optimized to entropy, and nonuniform optimal quantizer. The nonuniform quantizer has equal frequency of occurence for each quantized value in the quantized digital image [15].

An ideal entropy coder has been assumed. The independent coding of each pixel has been done, by an ideal entropy coder, which had the bit rate equal to the first order entropy of an image. One can expect a slight performance degradation when applying realizable entropy coders (Huffman, arithmetic, LZW).

The first group of results includes the fullband coding (FBC) of the image which will be compared to the SBC. We designed two types of quantizers in DPCM loop: a) the optimal quantizers (OQ) subject to the MSE constraint, and b) the uniform quantizers (UQ), followed by an entropy coder. Our experiment was carried outunder the condition that the number of levels of an OQ is the closest number to the first order entropy of the image at the uniform quantizer output. Let UQ(N)and OQ(N) represent an uniform and an optimal quantizer with $N = 2^n$ levels, respectively. The first order entropies in the case of UQ(8) and UQ(4) are calculatted to be 4. 41 and 2. 07 bpp, respectively.

(a)Test image Lena (256×256×8)

(b)Subband images ($M=4$) of test image Lena

Fig. 4

That is the reason why we have chosen OQ(5), OQ(4) and OQ(2).

In order to find the redundancy reduction R for FBC, the mean-square values of the quantizing noise σ_q^2 are calculated for the optimal quantizers and are given in Tab. 2. The calculation is carried out as the example for OQ(5). Taking into account the equation (5), we obtain $\sigma_e^2 = 161$. For Laplacian quantizer input signals, using the equation (9) $H_i = 5.60$ bpp, while the uniform quantizer stepsize is $\Delta = (2^8 \times 2) / 2^5 = 16$. Thus, from the equation (8), the corresponding output entropy will be $H = 5.60 - 4 = 1.60$ bpp, while the redundancy reduction R becomes $R = \log_2 n - H = 2.32 - 1.60 = 0.72$ bpp. The corresponding SQNR gain is $6.02 \times 0.72 = 4.33$ dB. On the other hand, the theoretical redundancy reduction bound equals SQNR gain / 6.02 = 5.6 / 6.02 = 0.93 bpp.

Table. 2 Simulation results for fullband EC-DPCM coding of test image lena

QUANTIZER	σ_q^2	σ_e^2	SQNR GAIN [dB]	REDUNDANCY REDUCTION [bpp]
OQ(5)	29	161	4.33	0.72
OQ(4)	104	370	4.82	0.80
OQ(2)	1730	1538	4.65	0.77

The second group of experiments introduces the 2-D SBC of the test image. The splitting scheme was used with the QMF filters with 16 taps, depicted as 16A[15]. The QMF bank was carried out in the separable, parallel structure with one stage. The filtering was performed in the Discrete Fourier Transform (DFT)frequency domain. For $M = 4$ subbands, we have the first-order entropies H_{11}, H_{12}, H_{21} and H_{22} of the subbands "11","12","21",and"22"respectively. The subimages for $M = 4$ are shown in Fig. 4b. They are measured to be 4.50; 2.55; 3.36 and 3.09 bpp in those subbands. The summary of the redundancy reduction results for SBC with $M = 4$ and $M = 7$ subbands are given in Tab. 3. a, b. respectively. The similar calculation. as previously stated for FBC. is carried out for the redundancy reduction in each subband. Taking into account the obtained values. R for SBC can be calculated by averaging the redundancy reduction of all subbands. Hence. it becomes 0.80 bpp. while SQNR gain = 4.82 dB.

The practical case $M = 7$ has another spectral subdivision in the lowest subband "11" on 4 new subbands: "11-11", "11-12"," 11-21" and " 11-22". The entropies H_{12}, H_{21} and H_{22} remain the same like in the former division for $M = 4$ sub-

Table. 3 Simulation results for subband EC-DPCM coding of test image lena

(a) $M=4$

SUBBAND	QUANTIZER	σ_q^2	σ_e^2	SQNR GAIN [dB]	REDUNDANCY REDUCTION [bpp]
11	OQ(5)	54	300	1.63	0.27
12	OQ(3)	288	576	6.38	1.06
21	OQ(4)	130	462	3.85	0.64
22	OQ(3)	228	456	7.40	1.23

(b) $M=7$

SUBBAND	QUANTIZER	σ_q^2	σ_e^2	SQNR GAIN [dB]	REDUNDANCY REDUCTION [bpp]
11-11	OQ(6)	0.62	34.96	14.99	2.49
11-12	OQ(3)	202	404	7.95	1.32
11-21	OQ(4)	90	320	5.42	0.90
11-22	OQ(3)	210	420	7.77	1.29

bands. $H_{11\text{-}11}$, $H_{11\text{-}12}$, $H_{11\text{-}21}$ and $H_{11\text{-}22}$ are 1.83; 2.65; 2.52 and 2.63 bpp, respectively. The obtained values give the redundancy reduction of 1.10 bpp, while the SQNR gain is 6.62 dB.

Also, we carried out the experiments in the case $M = 16$ subbands, as previously stated. Finally, Fig. 4, curve c, represents the SQNR gains in EC-DPCM subband system for $M = 4$, 7 and 16 subbands of the test image "Lena". As it can be seen, the practical SBC performance increament for the test image "Lena" is slower than the estimated theoretical upper bound on SQNR gain.

7 CONCLUSIONS

Starting from our previously stated results in the entropy coded DPCM literature, concerning an efficient fullband image coding, we have tried to combine subband coding ideas with entropy coded DPCM technique. A method for determining a bound of the coder performance efficiency based on the concept of the rate-distortion theory is developed in a new form. The performance results obtained after optimizing the EC-DPCM system parameters are very encouraging as they suggest performance improvements over DPCM as a method to encode subbands.

Bits are optimally allocated among the subbands to minimize the MSE for DPCM coding of the subbands. Optimum quantization is employed with quantizers matched to the Laplacian distribution, subject to the MSE constraint in the fullband DPCM system. In order to obtain the SQNR gain in a SBC EC-DPCM over fullband DPCM, we take into account the first-order entropy of the quantizer output and the allocated bit rate.

Finally, to obtain an upper bound on redundancy reduction for SBC system with EC-DPCM, we have to assume the minimum bit rate in $R(D)$ sense of the quantizer output for each subband. We obtained that the upper bound on redundancy reduction in the form which depends only on a number of subbands. As a result, we can achieve SQNR of 6.65 dB greater than a DPCM system for $M = 4$ subbands, while for $M = 16$, this gain exceeds 12 dB. This is an upper bound on SQNR gain over DPCM. The corresponding upper bound on redundancy reduction is 1.10 bits per pixel, as well as 2.10 bits per pixel for $M = 4$ and $M = 16$ subbands, respectively.

Simulation results prove that with subband number increament the redundancy reduction of still test image becomes greater. However, this performance increament for the still test image is slower than the upper bound on redundancy reduction, assuming the Laplacian pdf of quantizer Input.

Possible avenue for further research includes the study of an entropy encoder choice, as well as the EC-DPCM performance for noisy channels.

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熵子带图象编码中冗余量降低之上限的预测

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摘　要　本文讨论了最佳熵和均方差约束无记忆型量化器静止图象子带编码系统中性能效率的理论和实际估算。利用一失真理论概念推导出了一种特殊形式的函数，从而给出了用信噪比和冗余减少量表达的性能改善。在子带数 M 为 4，7 和 16 时，熵编码比特压缩率的理论上限为 1.10，1.51 和 2.10 比特。曾在测试图中演示并说明了实际性能的模拟结果。